YO-YO PHYSICS:

AN ENGINEER'S NOTEBOOK

MECHANICS
AND
GYROSCOPICS

MONOGRAPH IV
IN A SERIES

Don Watson

10/2001

Captain Yo

"ART IS LONG, LIFE SHORT;
JUDGMENT DIFFICULT,
OPPORTUNITY FLEETING."

JOHANN VON GOETHE
1749-1832

FIND FUN STUFF TO DO ALREADY -

D. WATSON
1924 -

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INTRODUCTION

IN 1984, AMERICAN SCIENTIST MAGAZINE PUBLISHED "THE YO-YO: A TOY FLYWHEEL" — A RARE ARTICLE IN ITS GENRE — BY WOLFGANG BÜRGER, PROFESSOR OF THEORETICAL MECHANICS AT THE UNIVERSITY OF KARLSRUHE, FEDERAL REPUBLIC OF GERMANY. THE ARTICLE, WITH INTRIGUING INSIGHTFUL CONTENT INCLUDING HIGHER CALCULUS BEYOND MY KEN, PROVIDED INSPIRATION FOR THIS MONOGRAPH SERIES AND PRESENTED A CHALLENGE TO STUDY AND LEARN MORE ABOUT YO-YO PHYSICS.

MONOGRAPHS I AND II DEVELOPED METHODS TO PREDICT AND DETERMINE YO-YO MOMENT OF INERTIA. MONOGRAPH III USED KNOWN MOMENT OF INERTIA IN ANALYZING THE PHYSICS OF "THE SLEEPING YO-YO"; IT MUST BE KNOWN FOR MOST OF MONOGRAPH IV AS WELL. HERE, SOME WORK OF DR. BÜRGER IS ADAPTED; SEE PART I, "TOY FLYWHEEL REVISITED", pgs. 3-11. ANCILLARY WORK FOLLOWS (pgs. 12-24) EXPANDING THE STUDY TO SUBJECTS OF USEFUL FURTHER INTEREST: VELOCITY OF FALL VS TIME, ACCELERATION, ANGULAR VELOCITY (RPM), etc.

PART II, "A TRUE GYROSCOPE" ANALYZES YO-YO GYROSCOPIC ACTION WITH AN INTUITIVE VIEW, THE IMPORTANT RIGHT HAND RULE, AND DEFINITION OF THE VECTOR COMPONENTS. THE OBJECTIVE IS TO DEVELOP A PRACTICAL UNDERSTANDING OF GYROSCOPE PHYSICS; ESPECIALLY OF PRECESSION, THAT UNIQUE AND UNEXPECTED TILTING OF THE

SPINNING DISK UNDER ANY TORQUE ATTEMPTING TO CHANGE THE ANGULAR POSITION OF THE SPIN AXIS. THE PRINCIPLES OF PRECESSION PHYSICS ARE THEN APPLIED TO THE YO-YO; FIRST, INVESTIGATING THE SLOW TILTING OF THE SLEEPING YO-YO WITH APPLIED STRING-TWIST TORQUE; THEN CALCULATING THAT RATE OF TILT (DEGREES PER SECOND) IN A COMMON YO-YO SITUATION.

TO MY TEACHERS IN ENGINEERING OF A HALF-CENTURY AGO, I OWE MY APPRECIATION FOR THEIR SKILL AND PATIENCE. OF EQUAL IMPORTANCE IS THE WORK OF ALL LISTED IN THE REFERENCES — MY NEWER TEACHERS: WOLFGANG BÜRGER AS ALREADY MENTIONED; HALLIDAY AND RESNICK, AND YOUNG AND FREEDMAN, FOR SIMILAR GUIDANCE IN NEWTONIAN MECHANICS; J.P. DEN HARTOG AND BANESH HOFFMANN FOR CLEAR TEACHING IN GYROSCOPE VECTOR AND MATHEMATICS PRINCIPLES; AND JOHN ARCHIBALD WHEELER FOR AN INTERESTING VIEW OF THE GYROSCOPE IN MODERN SPACE STUDIES — SEE HIS BOOK FOR THAT.

CALCULATIONS HERE WERE PERFORMED WITH AN INEXPENSIVE (TI-60) CALCULATOR, WITH SOME TABLES CHECKED BY COMPUTER SPREADSHEET. FOR BEST UNDERSTANDING: IN PLACES, READ AND COGITATE; IN OTHERS, SCRIBBLE AND CALCULATE. MUCH EFFORT HAS BEEN EXPENDED, CHECKING AND RECHECKING. ERRORS IN LOGIC, TEXT, OR NUMBERS THAT MIGHT REMAIN ARE MY OWN.

MORE FUN WITH THE YO-YO FOLLOWS . . .

PART I. "A TOY FLYWHEEL" REVISITED

IN PROFESSOR BÖRGER'S MAGAZINE ARTICLE, A "LONG SPIN" YO-YO WAS USED FOR ANALYSIS. MANY OF THE SAME CHARACTERISTICS (AXLE AND STRING SPOOL RADIUS, STRING LENGTH, AND RADIUS OF GYRATION) ARE USED HERE IN A FICTICIOUS BUT PRACTICAL "PLAYABLE YO-YO". WEIGHT AND MOMENT WERE NOT DEFINED IN THE ARTICLE; FOR THOSE, DEFINITIONS CONSISTENT WITH THE RADIUS OF GYRATION ARE GIVEN HERE (SEE pg. 7).

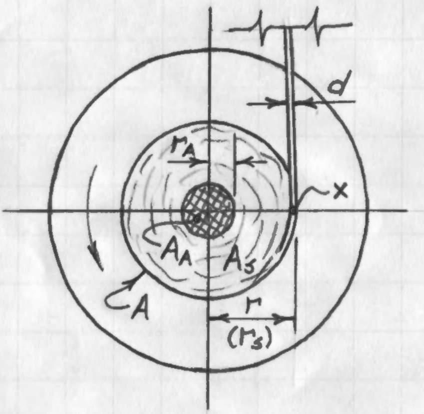
QUOTING FROM THE ARTICLE: "... THE YO-YO REACHES A MAXIMUM VELOCITY ABOUT HALFWAY DOWN THE STRING...". WHILE NOT IN QUESTION, THE STATEMENT IS FULLY SUPPORTED HERE; SEE "VERTICAL VELOCITY-DISTANCE", pg 11.

PART I CONTINUES WITH SUBJECTS OF FURTHER INTEREST, PRESENTED IN DEGREES OF DEPTH AND DETAIL NOT SUITABLE FOR MAGAZINE PUBLICATION. GRAPHIC TRANSFORMATION OF DATA IS USED TO GENERATE THE "VERTICAL VELOCITY-TIME" DATA AND GRAPH; A METHOD ALTERNATIVE TO ADVANCED CALCULUS IN DETERMINING YO-YO TIME OF FALL. ACCELERATION, LEVERAGE AND TENSION EFFECTS, ANGULAR VELOCITY-RPM, AND CONSERVATION OF ENERGY COMPLETE PART I.

READERS: PLAYERS, MAY WHAT FOLLOWS PROVIDE INTEREST AND INSIGHTS TO IMPROVE YOUR SKILLS; NON-PLAYERS, MAY WHAT FOLLOWS PROVIDE INTEREST AND INSIGHTS TO BECOME A SKILLED PLAYER.

EFFECTIVE STRING THICKNESS

A PLAYABLE YO-YO HAS A STRING GAP WIDER THAN THE STRING THICKNESS. MORE STRING TURNS CAN BE "PACKED" IN THE STRING GAP THAN IF GAP WIDTH AND STRING THICKNESS WERE EQUAL. STUDY OF MOTION IN A DROP OR THROW OF THE YO-YO FROM THE HAND MUST TAKE ACCOUNT OF THE EFFECTIVE STRING THICKNESS - NOT DIRECTLY MEASURABLE. FROM THE SKETCH, WITH COMMON VALUES:



r = FULLY WOUND STRING SPOOL RADIUS, 1.3 cm

r_A = AXLE RADIUS, 0.3 cm

N = TURNS IN THE FULLY WOUND STRING, 20

$$d = \frac{r - r_A}{N} = \frac{1.3 - 0.3}{20}\ \text{cm}$$

$$d = 0.050\ \text{cm} \quad (\simeq 0.020\ \text{in})$$

COMMON MODERN STRING GAPS ARE IN THE RANGE FROM 0.080" TO 0.120", WHILE MOST COMMON YO-YO STRING THICKNESSES ARE FROM 0.050" TO 0.060".

IN THE SKETCH, THE TURNS N IN THE FULLY WOUND STRING SPOOL BEGIN AT THE AXLE AND END AT THE POINT "X". TYPICALLY, YO-YO STRING IS USED AT EFFECTIVE LENGTH $L \simeq 39$ in $\simeq 1.0$ m.

DECREASING SPOOL RADIUS

THE "ACADEMIC YO-YO" OF MONOGRAPH II REQUIRED THAT THE SPOOL RADIUS r_s BE HELD CONSTANT — ACCOMPLISHED USING A VERY THIN FILAMENT WOUND IN A VERY WIDE STRING GAP.

MOTION ANALYSIS FOR A PLAYABLE YO-YO ALLOWS NO SUCH MODIFICATIONS. HERE, FULL ACCOUNT MUST BE TAKEN OF THE VARIATION (DECREASE) IN SPOOL RADIUS AS THE STRING UNWINDS ALONG ITS FULL LENGTH L. LET:

L = YO-YO STRING LENGTH, 1 m = 100 cm
s = UNWOUND PART OF STRING, cm

DEFINING THREE AREAS FROM THE SKETCH:

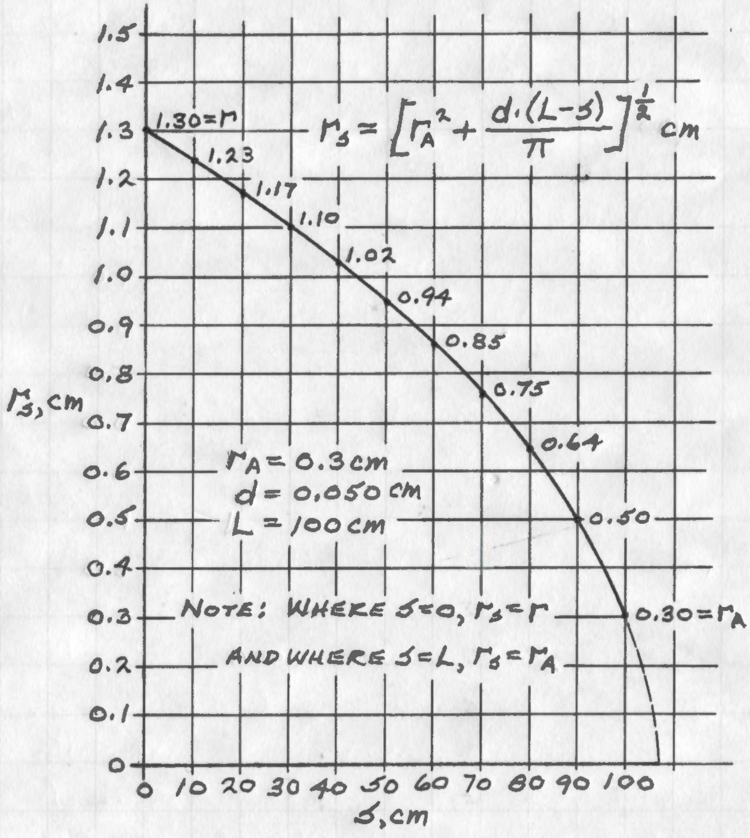
A = AXLE AREA AND STRING COVERED AREA FOR ANY s COMBINED, πr_s^2 cm^2
A_A = AXLE AREA ALONE, πr_A^2 cm^2
A_s = STRING SPOOL ANNULAR AREA FOR ANY s, $d \cdot (L-s)$ cm^2; AT $s=0$, $A_s = d \cdot L$

FOR ANY s; $A = A_A + A_s$:

$$\pi r_s^2 = \pi r_A^2 + [d \cdot (L-s)] \text{ cm}^2$$

$$r_s = \left[r_A^2 + \frac{d \cdot (L-s)}{\pi} \right]^{\frac{1}{2}} \text{ cm}$$

r_s IS A DECREASING FUNCTION OF s, GIVEN r_A, d, L, AND π ARE KNOWN CONSTANTS. r_s IS EASILY PLOTTED AGAINST s USING COMMON PLAYABLE YO-YO VALUES:



CLEARLY, r_s DECREASES MOST RAPIDLY AS THE YO-YO NEARS THE END OF THE STRING WHERE $r = r_A = 0.3$ cm, EQUIVALENT TO AN AXLE DIAMETER OF ABOUT 1/4 in.

AS WILL BE SEEN, THE EQUATION DEFINING r_s CAN BE USED TO DETERMINE VERTICAL VELOCITY OF THE YO-YO FALLING ALONG STRING LENGTH L.

VERTICAL VELOCITY OF FALL

FREE BODY ANALYSIS OF THE "ACADEMIC YO-YO" (MONOGRAPH II, pg. 7) GIVES THE VERTICAL ACCELERATION a OF A FALLING YO-YO WITH FIXED SPOOL RADIUS r AS:

$$a = g\left[\frac{1}{1+(J/Mr^2)}\right]\frac{m}{sec^2}$$

WITH r CONSTANT, a IS CONSTANT. THE VELOCITY V OF A BODY STARTING FROM REST AND SUBJECT TO A CONSTANT ACCELERATION a APPLIED THROUGH A GIVEN DISTANCE S IS:

$$V = (2aS)^{\frac{1}{2}}\ \frac{m}{sec}$$

IN A PLAYABLE YO-YO (WHERE r DECREASES AS S INCREASES), COMBINING THE EQUATIONS ELIMINATES THE CONSTANT a IN DEFINING V_S AS A FUNCTION OF CONSTANTS g, J, M, AND THE TWO RELATED VARIABLES, S AND r_S:

$$V_S = \left[\frac{2gS}{1+(J/Mr_S^2)}\right]^{\frac{1}{2}}\frac{m}{sec}$$

THE TABLE FOLLOWING GIVES CALCULATED VALUES FOR V_S USING $J = 28125\times10^{-9}\,kg\text{-}m^2$ AND $M = 45\times10^{-3}\,kg$ FOR A TYPICAL YO-YO DESIGN. S IS GIVEN IN 10 cm (0.1 m) INCREMENTS FOR A TYPICAL STRING LENGTH L OF 1 m.

VERTICAL VELOCITY - DEFINITIONS

VELOCITY OF FALL VARIABLES FOR THE PLAYABLE YO-YO AND ACADEMIC YO-YO ARE HERE DEFINED FOR THE DATA TABLE. BY COLUMN NUMBER:

1. S, m. UNWOUND STRING IN 0.1m INCREMENTS FROM 0 TO 1.0 m.

PLAYABLE YO-YO (d = 0.050 cm):

2. r_s, m$\times10^{-3}$. STRING SPOOL RADIUS AT EACH 0.1m IN S; SEE "DECREASING SPOOL RADIUS".
3. V_s, m/sec. SEE "VERTICAL VELOCITY OF FALL" WHERE $g = 9.81$ m/sec^2.
4. AVG. V_s, m/sec. $(V_{s-1} + V_s)/2$ AT EACH INTERVAL.
5. t_s, sec. INTERVAL DURATION; 0.1m / AVG. $V_s \frac{m}{sec}$.
6. Cum. t_s, sec. CUMULATIVE t_s AT EACH INTERVAL; cum $t_{s-1} + t_s$. AT S=1.0m, TOTAL TIME OF FALL = Σt_s.

ACADEMIC YO-YO (d = 0 cm):

7. r_s, m$\times10^{-3}$. "STRING SPOOL" RADIUS, CONSTANT AT THE AXLE RADIUS; 3×10^{-3}m.
8, 9, 10, 11. EACH CALCULATED AS IN COLUMS 3 THROUGH 6, RESPECTIVELY.

THREE SIGNIFICANT FIGURES ARE REPORTED FOR MOST DATA TO MINIMIZE PROGRESSIVE LOSS OF ACCURACY, NOT TO CLAIM PRECISION AT THAT LEVEL.

TABLE 1: VELOCITY - DISTANCE

	PLAYABLE YO-YO					ACADEMIC YO-YO				
1	2	3	4	5	6	7	8	9	10	11
			INTERVAL					INTERVAL		
S, m	r_s, m×10^{-3}	V_s, m/sec	Avg. V_s	t_s, sec	Cum. t_s	r_s, m×10^{-3}	V_s, m/sec	Avg. V_s	t_s, sec	Cum. t_s
0.0	13.00	0.000	—	—	—	3.00	0.000	—	—	—
0.1	12.34	0.620	0.310	0.323	0.323	3.00	0.167	0.084	1.190	1.190
0.2	11.68	0.838	0.729	0.137	0.460	3.00	0.236	0.202	0.495	1.685
0.3	10.97	0.975	0.907	0.110	0.570	3.00	0.289	0.263	0.380	2.065
0.4	10.22	1.060	1.018	0.098	0.668	3.00	0.334	0.312	0.321	2.386
0.5	9.41	1.103	1.082	0.092	0.760	3.00	0.373	0.354	0.282	2.668
0.6	8.52	1.107	1.105	0.090	0.850	3.00	0.409	0.391	0.256	2.924
0.7	7.53	1.069	1.088	0.092	0.942	3.00	0.442	0.426	0.235	3.159
0.8	6.39	0.981	1.025	0.098	1.040	3.00	0.472	0.457	0.219	3.378
0.9	4.99	0.823	0.902	0.111	1.151	3.00	0.501	0.487	0.205	3.583
*$L = 1.0$ m	3.00	0.528	0.676	0.148	1.299	3.00	0.528	0.515	0.194	3.777

$\Sigma t_s = 1.299$ sec

*$T_F \simeq 1.3$ sec

$\Sigma t_s = 3.777$ sec

$T_F \simeq 3.8$ sec

*$\text{Avg. } V_s = \frac{L}{T_F} = \frac{1.0}{1.3} = 0.770 \frac{m}{sec}$

VERTICAL VELOCITY – DISTANCE

PLAYABLE YO-YO DATA V_s (COLUMN 3) PLOTTED AGAINST S (COLUMN 1) REVEALS V_s INITIALLY INCREASING BUT WITH DECLINING ACCELERATION, REACHING [1] "... MAXIMUM VELOCITY ABOUT HALF-WAY DOWN THE STRING". V_s THEN RECEDES AS THE ACCELERATION BECOMES INCREASINGLY NEGATIVE. THIS TRAJECTORY RESULTS FROM THE DECLINING STRING SPOOL RADIUS – DESPITE A LESS SIGNIFICANT INCREASE IN STRING TENSION. SEE "LEVERAGE AND TENSION EFFECTS".

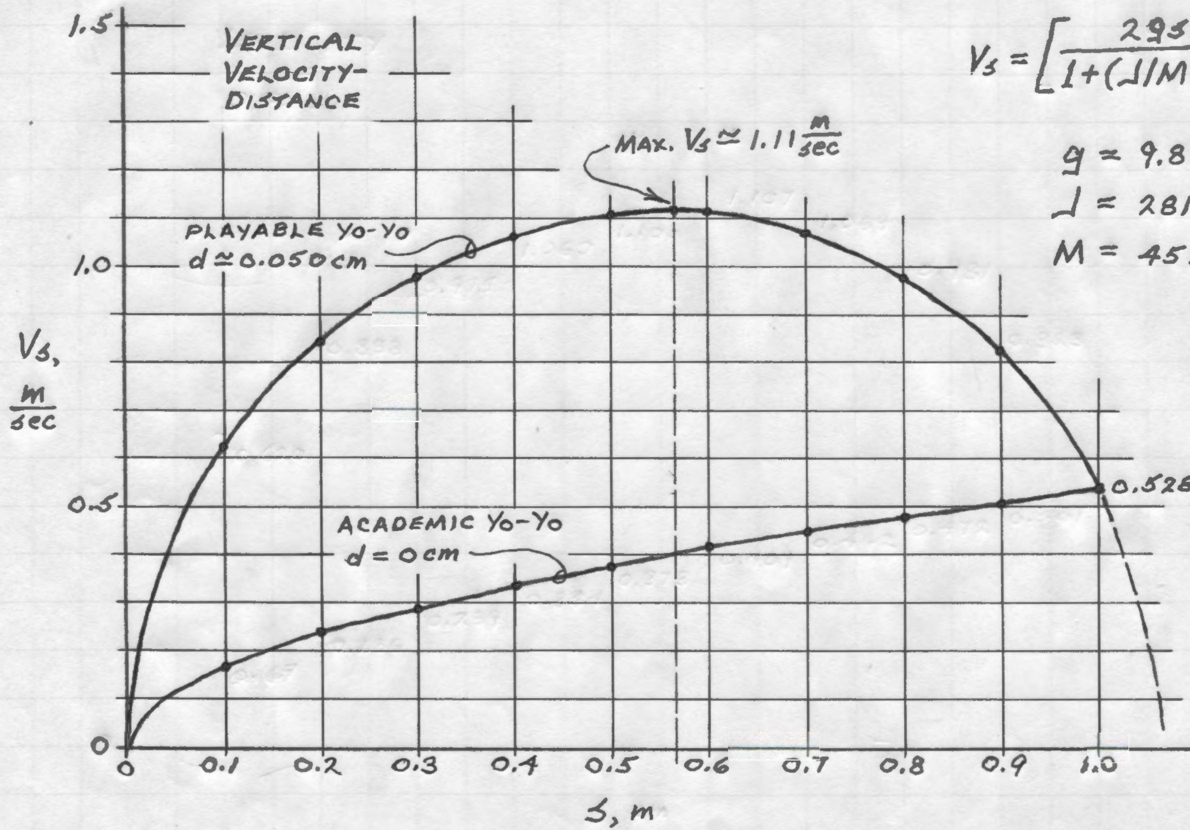
ACADEMIC YO-YO DATA SIMILARLY PLOTTED EXHIBITS A MUCH FLATTER TRAJECTORY. THE SQUARE ROOT FUNCTION IN THE V_s EQUATION (WITH S INCREASING IN EQUAL INCREMENTS AND r_s HELD CONSTANT) DICTATES THE PARABOLIC FORM IN THIS PLOT.

BOTH YO-YOS AT $S = 1.0$ m ARE ALSO AT $r_s = 0.3$ cm AND BOTH THERE REACH THE SAME VERTICAL VELOCITY $V_s \simeq 0.53$ m/sec. SEE "CONSERVATION OF ENERGY" FOR FURTHER INFORMATION.

TIME OF FALL t_s IN ANY INTERVAL IS DERIVED DIVIDING THE INTERVAL DISTANCE S (0.1 m) BY THE AVERAGE VELOCITY AVG. V_s FOR THE INTERVAL. THE TOTAL TIME OF FALL T_F THEN IS Σt_s.

NOTE THE ALMOST 300% GREATER FALL DURATION T_F FOR THE ACADEMIC YO-YO.

[1] SEE REFERENCES: "THE YO-YO...", BÜRGER.



$$V_s = \left[\frac{2gs}{1+(J/Mr_s^2)}\right]^{\frac{1}{2}} \frac{m}{sec}$$

$$g = 9.81 \frac{m}{sec^2}$$

$$J = 28125 \times 10^{-9}\, kg\text{-}m^2$$

$$M = 45 \times 10^{-3}\, kg$$

VERTICAL VELOCITY-TIME

THE VELOCITY-DISTANCE DATA TABLE 1 DEFINES THE PLAYABLE YO-YO FALL TIME $T_F = 1.3$ sec. FOR THE TEN 0.1 m DISTANCE INTERVALS. DATA FOR S, CUM t_s, AND V_s ARE COPIED TO VELOCITY-TIME DATA TABLE 2 WHERE COLUMNS 13 AND 14 IDENTIFY TEN "POINTS PLOTTED ⊗". THOSE POINTS PLOTTED ON A THIRTEEN 0.1 sec INTERVAL TIME BASE DEFINE THE VERTICAL VELOCITY-TIME CURVE. FROM THIS NEW CURVE, "POINTS READ ⊙" DEFINE t AND V_t DATA FOR THIRTEEN 0.1 sec. TIME INTERVALS; SEE COLUMNS 15 AND 16.

COLUMN 17 DEVELOPS AVG. $V_t = (V_{n-1} + V_n)/2$ FOR EACH INTERVAL. AT COLUMN 18, THE YO-YO IS ESTIMATED TO FALL A DISTANCE $S_t = V_{AVG.} \cdot t$ m FOR EACH INTERVAL; THE TOTAL $L = \Sigma S_t \simeq 1.0096$ m IS WITHIN 1% OF THE KNOWN 1.0 m STRING LENGTH. A SHADED AREA UNDER THE VELOCITY-TIME CURVE DEFINES $S_T = 0.050$ m IN THE THIRD TIME INTERVAL (WITH DATA OUTLINED IN TABLE 2). SUMMATION OF THE THIRTEEN SUCH AREAS DETERMINE THE GRAPHIC INTEGRAL $\Sigma S_t \simeq 1.0096$ m.

THE TOTAL AREA ΣS_t DIVIDED BY ITS BASE T_F YIELDS THE OVER-ALL AVERAGE VELOCITY OF FALL AVG. $V_t = \Sigma S_t / T_F = 1.0096\text{ m}/1.3\text{ sec} \simeq 0.78$ m/sec.

VELOCITY-DISTANCE TO VELOCITY-TIME CONVERSION BY THIS GRAPHIC METHOD GIVES RESULTS OF QUITE ACCEPTABLE ACCURACY IN THIS ANALYSIS.

TABLE 2: PLAYABLE YO-YO DATA

VERTICAL VELOCITY-TIME POINTS PLOTTED ⊗ | POINTS READ ⊙ | ACCELERATION-TIME:

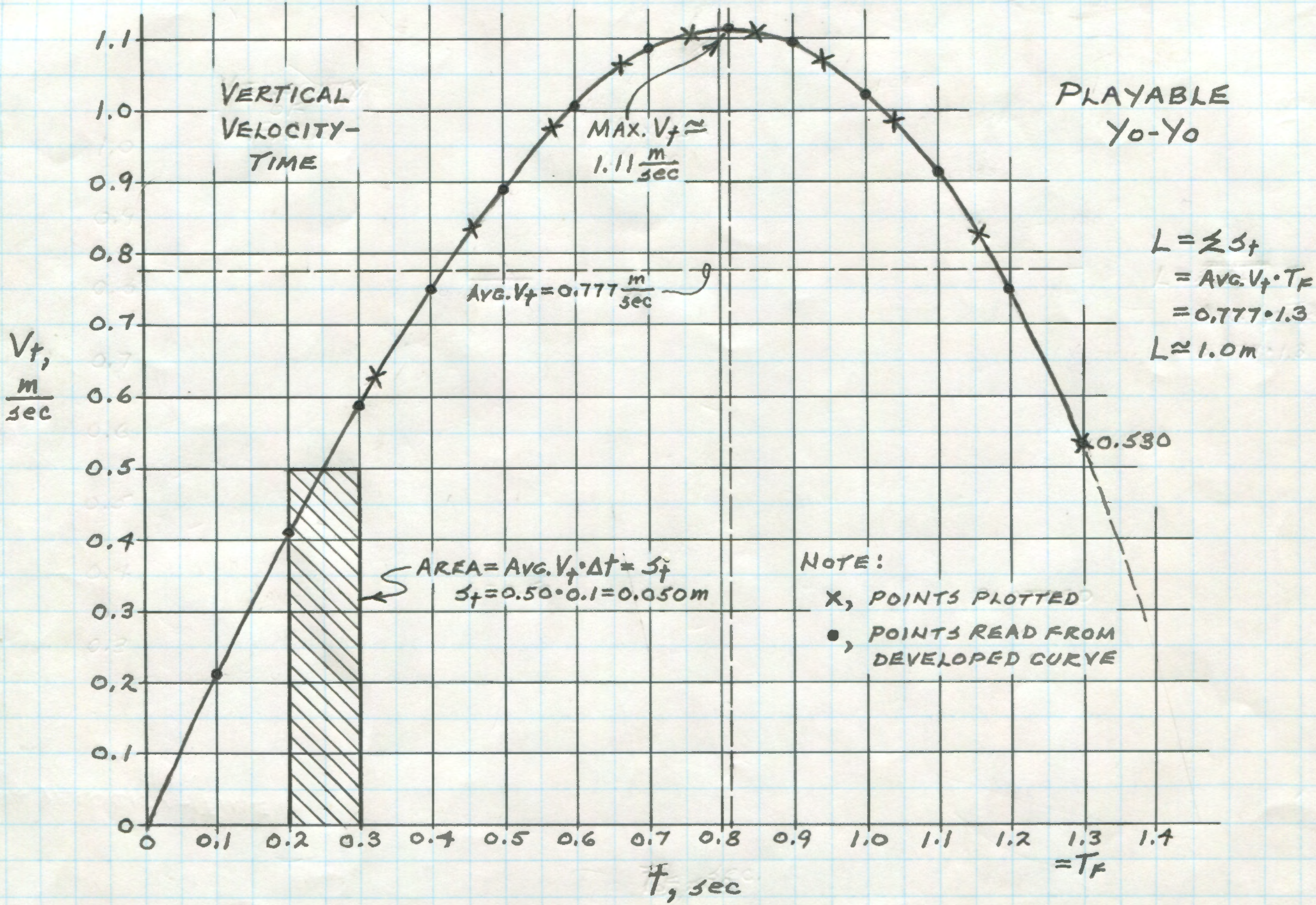
$$a_t = \frac{\Delta V_t}{\Delta t}\,\frac{\frac{m}{sec}}{sec} = \frac{\Delta V_t}{0.1}\,\frac{m}{sec^2}$$

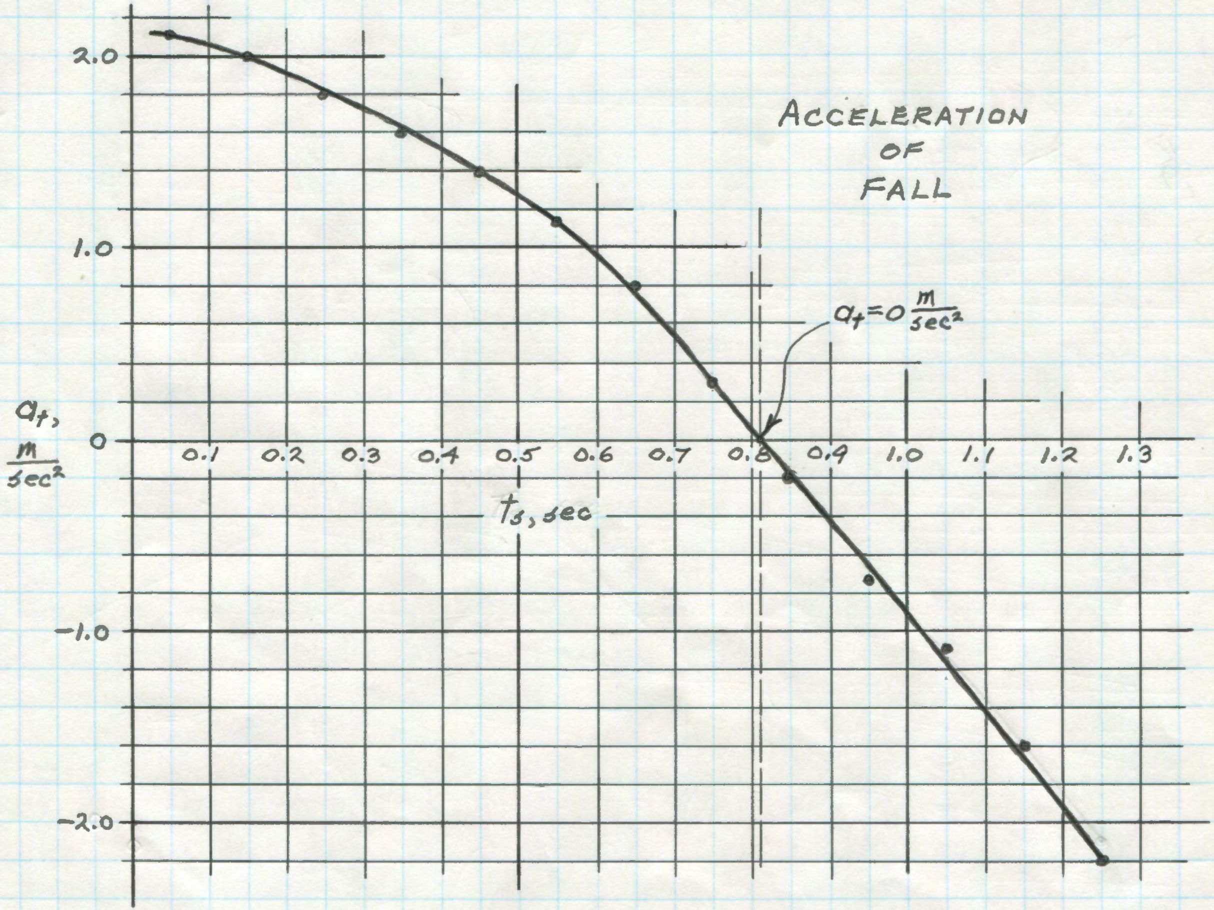
12	13	14	15	16	17	18	19	20
s, m	Cum. t_s, sec	V_s, m/sec	t, sec	V_t, m/sec	Avg. V_t	s_t, m	ΔV_t, m/sec	a_t m/sec^2
0.0	—	0.000	0.0		—	—	—	—
0.1	0.323	0.620	0.1	0.210	0.105	0.0105	0.210	2.10
0.2	0.460	0.838	0.2	$\boxed{0.410}$	0.310	0.0310	0.200	2.00
0.3	0.570	0.975	0.3	$\boxed{0.590}$	$\boxed{0.500}$	$\boxed{0.0500}$	0.180	1.80
0.4	0.668	1.060	0.4	0.750	0.670	0.0670	0.160	1.60
0.5	0.760	1.103	0.5	0.890	0.820	0.0820	0.140	1.40
0.6	0.850	1.107	0.6	1.005	0.948	0.0948	0.115	1.15
0.7	0.942	1.069	0.7	1.085	1.045	0.1045	0.080	0.80
0.8	1.040	0.981	0.8	1.115	1.100	0.1100	0.030	0.30
0.9	1.151	0.823	0.9	1.095	1.105	0.1105	−0.020	−0.20
*L = 1.0 m	1.299	0.528	1.0	1.020	1.058	0.1058	−0.075	−0.75
			1.1	0.910	0.965	0.0965	−0.110	−1.10
			1.2	0.750	0.830	0.0830	−0.160	−1.60
			*T_F = 1.3 sec	0.530	0.640	0.0640	−0.220	−2.20

*$L = \Sigma s_t \simeq 1.0096\,m$

*ERROR IN $\Sigma s_t < 1\%$

$$\text{AVG. } V_t = \frac{L}{T_F} = \frac{1.0096}{1.3} = 0.777\,\frac{m}{sec}$$


VERTICAL VELOCITY-TIME
PLAYABLE YO-YO
MAX. $V_t \approx 1.11 \frac{m}{sec}$
AVG. $V_t = 0.777 \frac{m}{sec}$
$L = \Sigma s_t$
$= \text{AVG.} V_t \cdot T_F$
$= 0.777 \cdot 1.3$
$L \approx 1.0 m$
0.530
AREA = AVG. $V_t \cdot \Delta t = s_t$
$s_t = 0.50 \cdot 0.1 = 0.050 m$
NOTE:
X, POINTS PLOTTED
●, POINTS READ FROM DEVELOPED CURVE
V_t, $\frac{m}{sec}$
t, sec
$= T_F$
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4


ACCELERATION
OF
FALL
$a_t = 0 \frac{m}{sec^2}$
a_t, $\frac{m}{sec^2}$
2.0
1.0
0
-1.0
-2.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
t_s, sec

ACCELERATION-TIME

INITIAL VELOCITY FOR AN INTERVAL AND ACCELERATION DURING THE INTERVAL CAN BE USED TO DETERMINE THE YO-YO FALL DISTANCE. TABLE 2 RESULTS (FROM $S_t = V_{AVG.} \cdot t$) CAN BE CHECKED WITH THE ALTERNATIVE APPROACH:

$$S_t = (V_o \cdot t) + (\tfrac{1}{2} \cdot a_t \cdot t^2); \quad t = 0.1 \text{ sec}$$

INTERVAL	[1] $V_o \cdot t$	a_t	[2] $(\frac{1}{2} \cdot a_t \cdot t^2)$	S_t, m
1	0.0000	2.10	0.0105	0.0105
2	0.0210	2.00	0.0100	0.0310
3	0.0410	1.80	0.0090	0.0500
4	0.0590	1.60	0.0080	0.0670
5	0.0750	1.40	0.0070	0.0820
6	0.0890	1.15	0.0058	0.0948
7	0.1005	0.80	0.0040	0.1045
8	0.1085	0.30	0.0015	0.1100
9	0.1115	-0.20	-0.0010	0.1105
10	0.1095	-0.75	-0.0038	0.1057
11	0.1020	-1.10	-0.0055	0.0965
12	0.0910	-1.60	-0.0080	0.0830
13	0.0750	-2.20	-0.0110	0.0640
14	—		$L = \Sigma S_t \approx$	1.0095

[1] SEE VERTICAL VELOCITY-TIME CURVE FOR V_o AT EACH INTERVAL.

[2] SEE ACCELERATION OF FALL CURVE FOR ACCELERATION IN EACH INTERVAL.

LEVERAGE AND TENSION EFFECTS

THE PLAYABLE YO-YO WITH ITS MOMENT OF INERTIA $I = 28125 \times 10^{-9}\,kg\text{-}m^2$ AND WEIGHT $M = 45 \times 10^{-3}\,kg$ EXHIBITS A RADIUS OF GYRATION k_o OF:

$$k_o = \left(\frac{I}{M}\right)^{\frac{1}{2}} = \left(\frac{28125 \times 10^{-9}}{45 \times 10^{-3}}\right)^{\frac{1}{2}} = (625 \times 10^{-6})^{\frac{1}{2}} = 25 \times 10^{-3}\,m$$

$$k_o = 2.5\,cm$$

MONOGRAPH I (pages 4 AND 5) DEFINED STRING TO YO-YO LEVERAGE AS $L\% = \frac{r}{k_o} \times 100\%$. IN THE PLAYABLE YO-YO, r_s VARIES FROM $r_s = r = 1.3\,cm$ AT RELEASE TO $r_s = r_A = 0.3\,cm$ AT THE END OF THE FALL. INITIAL LEVERAGE HERE IS MORE THAN FOUR TIMES THE FINAL VALUE:

$$L_i\% = \frac{1.3}{2.5} \times 100\% = 52\% \text{ AND } L_f\% = \frac{0.3}{2.5} \times 100\% = 12\%$$

THIS VERY SIGNIFICANT AND CONTINUOUS LOSS OF LEVERAGE RESULTS IN AN INCREASING TENSION T IN THE STRING. TENSION T APPROACHES SOME VALUE LESS THAN THE YO-YO WEIGHT M AS r_s NEARS r_A. T MIGHT REACH M IF r_A WERE (IMPOSSIBLY) ZERO.

IN THE FREE BODY DIAGRAM, SUMMATION OF TORQUES $\Sigma\Upsilon$ YIELDS:

$$\Sigma\Upsilon = T\cdot r_s - J\cdot\alpha_s = 0; \quad T\cdot r_s = J\cdot\alpha_s$$

WITH $J = Mk_o^2$, AND $\alpha_s = \frac{a}{r_s}$:

$$T\cdot r_s = M\cdot k_o^2\cdot\frac{a}{r_s}$$

$$T = \frac{k_o^2}{r_s^2}\cdot M\cdot a$$

HERE, $a = \dfrac{g}{1+\left(\frac{J}{Mr_s^2}\right)}$, OR $a = \dfrac{g}{1+\left(\frac{\cancel{M}k_o^2}{\cancel{M}r_s^2}\right)}$

THEN, $T = \dfrac{k_o^2}{r_s^2}\cdot M\cdot\dfrac{g}{1+\left(\frac{k_o^2}{r_s^2}\right)}$

$$= \frac{k_o^2\cdot M\cdot g}{r_s^2+k_o^2} = \frac{M\cdot g}{\frac{r_s^2+k_o^2}{k_o^2}}$$

AND, AFTER ALL THIS ALGEBRA FUN:

$$T = \frac{Mg}{\left(\frac{r_s}{k_o}\right)^2+1}\ \frac{kg\text{-}m}{sec^2}, \text{ OR Newton}$$

THIS EQUATION DEFINES T AS THE GRAVITY FORCE Mg IN Newton UNITS REDUCED IN DIVISION BY (A DIMENSIONLESS FACTOR) "THE LEVERAGE RATIO SQUARED PLUS ONE". NOTE THAT AS r_s APPROACHES ZERO, T APPROACHES THE GRAVITY FORCE Mg OR WEIGHT OF THE YO-YO IN Newtons.

THE GRAVITY FORCE Mg AND YO-YO WEIGHT M ARE EQUIVALENT VALUES AS CAN BE SHOWN IN THE UNIT CONVERSION:

$$1\,N = 0.225\,lb \cdot 453.6\times10^{-3}\,\frac{kg}{lb} = 102.1\times10^{-3}\,kg$$

WITH $M = 45\times10^{-3}\,kg$ AND $g = 9.81\,\frac{m}{sec^2}$:

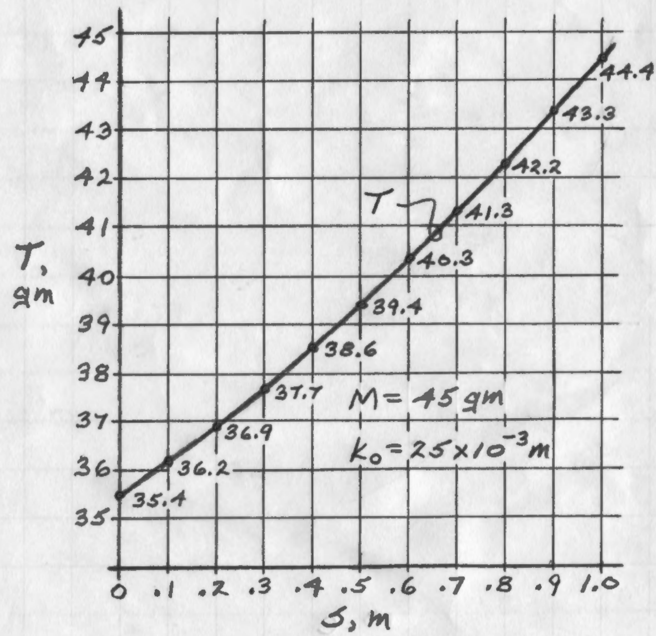
$$Mg = 0.441\,N \cdot 102.1\times10^{-3}\,\frac{kg}{N} = 45\times10^{-3}\,kg$$

IN THE UNAIDED GRAVITY INDUCED FALL, Mg IN Newtons CAN BE STATED AS M IN grams, AND:

$$T = \frac{M}{\left(\frac{r_s}{k_o}\right)^2 + 1}\ gm$$

USING $k_o = 25\times10^{-3}\,m$ AND PLOTTING T:

S, m	$r_s \times 10^{-3}$, m
0.0	13.0
0.1	12.3
0.2	11.7
0.3	11.0
0.4	10.2
0.5	9.4
0.6	8.5
0.7	7.5
0.8	6.4
0.9	5.0
1.0	3.0



THIS PLAYABLE YO-YO EXHIBITS A NOMINAL 25% INCREASE IN STRING TENSION OVER THE STRING LENGTH OF 1.0 M.

ANGULAR VELOCITY – RPM

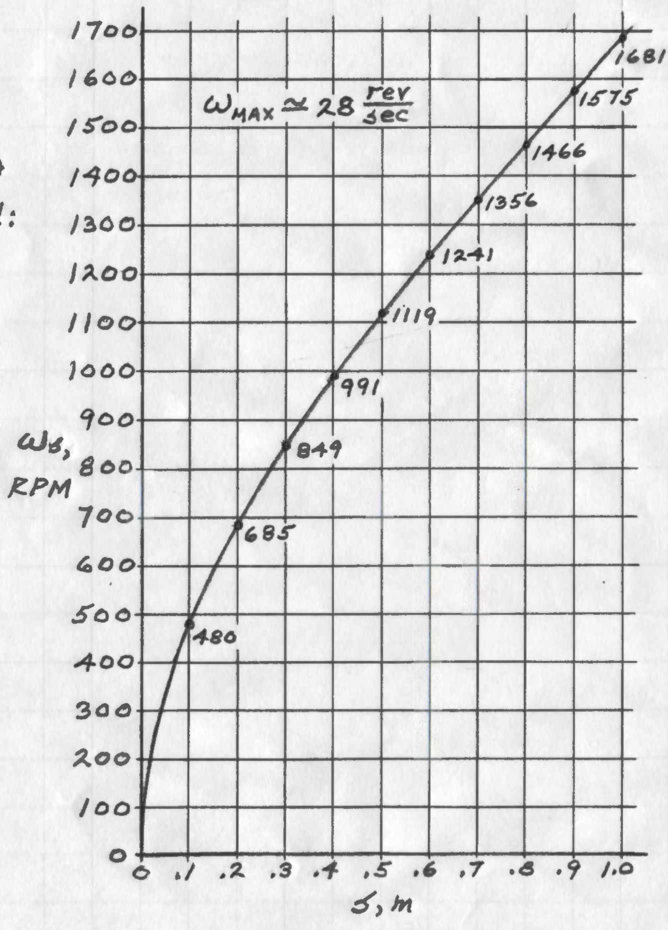
AT ANY POINT IN THE PLAYABLE YO-YO FALL, ITS ANGULAR VELOCITY ω_s IN RPM IS:

$$*\omega_s = \frac{V_s}{r_s}\frac{\cancel{rad}}{\cancel{sec}} \cdot \frac{1}{2\pi}\frac{rev}{\cancel{rad}} \cdot 60\frac{\cancel{sec}}{min}$$

$$\omega_s = \frac{30\,V_s}{\pi\, r_s}\ \text{RPM}$$

V_s AND r_s DATA FROM TABLE 1:

$V_s, \frac{m}{sec}$	$r_s, m \times 10^{-3}$
0.000	13.00
0.620	12.34
0.838	11.68
0.975	10.47
1.060	10.22
1.103	9.41
1.107	8.52
1.069	7.53
0.981	6.39
0.823	4.99
0.528	3.00



$$*\ \text{OR,}\ \ \omega_s = \left(V_s\frac{\cancel{m}}{\cancel{sec}} / 2\pi r_s \frac{\cancel{m}}{rev}\right)\cdot 60\frac{\cancel{sec}}{min} = \frac{30\,V_s}{\pi\, r_s}\ \text{RPM}$$

CONSERVATION OF ENERGY

[1]"ENERGY...CAN BE CONVERTED FROM ONE FORM TO ANOTHER BUT CANNOT BE CREATED OR DESTROYED." THE PLAYABLE YO-YO, READY TO RELEASE FROM THE HAND IN A GRAVITY-INDUCED FALL (1.0 m), HAS A POTENTIAL ENERGY P.E. OF 45 gm-m. IN THE FALL P.E. IS CONVERTED TO TRANSLATION ENERGY T.E. IN VERTICAL VELOCITY, AND ROTATION ENERGY R.E. IN ANGULAR VELOCITY ABOUT ITS OWN AXIS.

IGNORING FRICTION AND OTHER EFFECTS, INDUCED KINETIC K.E. AT ANY INSTANT IS THE SUM OF T.E. AND R.E., AND IS EQUAL TO THE P.E. EXPENDED UP TO THAT INSTANT. VERTICAL VELOCITY V_S AND ANGULAR VELOCITY ω_S AT EVERY INSTANT MUST DISPLAY THE K.E. TO P.E. EQUALITY.

TABLE 3 DEFINES ENERGY VALUES AND CONVERSIONS. NOTE THAT AT 1.0 m, P.E. AND K.E. EACH $\simeq$ 0.441 N-m WHERE P.E. WAS GIVEN AT 45 gm-m. ELIMINATING Newtons FROM N-m UNITS:

$$P.E. = K.E. \simeq 0.441\, N\text{-}m \cdot 0.225 \frac{lb}{N} \cdot 453.6 \frac{gm}{lb}$$

$$P.E. = K.E. \simeq 45\, gm\text{-}m$$

THE ENERGY CONVERSION CURVES PROVE THE YO-YO VERY EFFICIENT IN THE CONVERSION OF POTENTIAL ENERGY TO ROTATIONAL ENERGY.

[1] SEE REFERENCES: 3, "UNIVERSITY PHYSICS".

TABLE 3: CONSERVATION OF ENERGY

PLAYABLE YO-YO

$$P.E. = Mgs = kg \cdot \frac{m}{sec^2} \cdot m = \frac{kg\text{-}m^2}{sec^2} = N\text{-}m$$

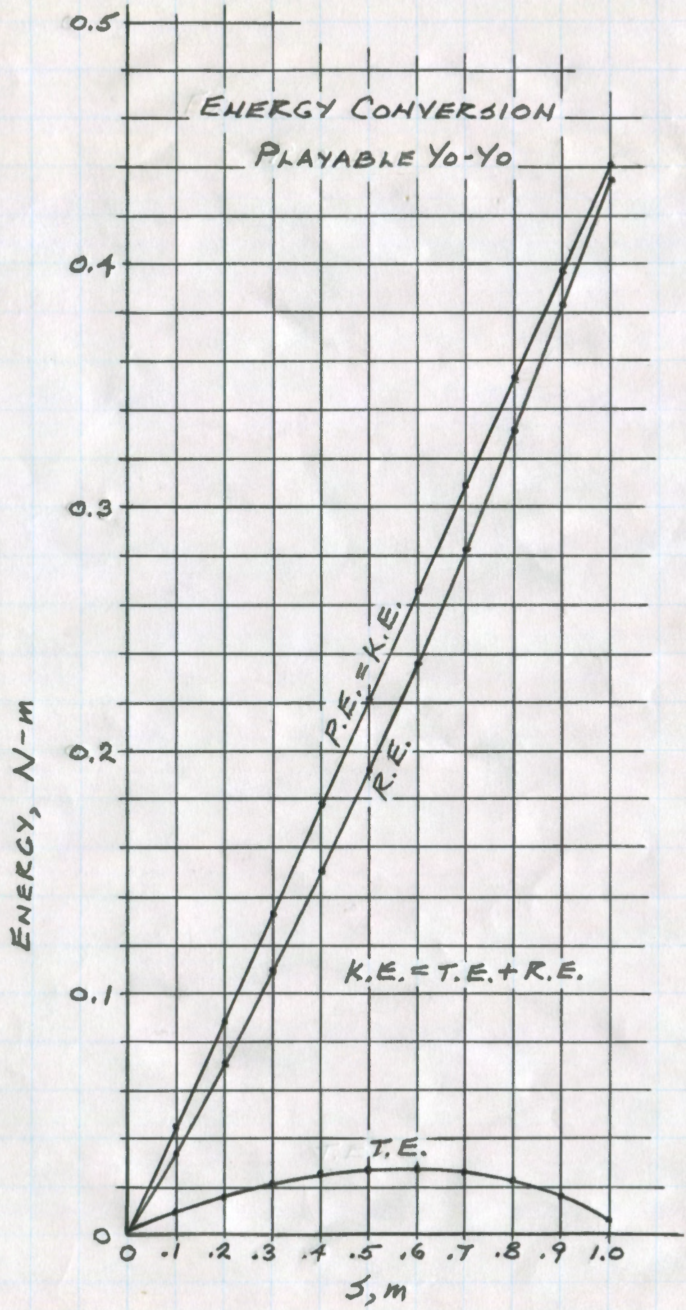
$$T.E. = \frac{1}{2}MV_s^2 = kg \cdot \frac{m^2}{sec^2} = \frac{kg\text{-}m^2}{sec^2} = N\text{-}m$$

$$R.E. = \frac{1}{2}I\omega_s^2 = kg\text{-}m^2 \cdot \frac{\cancel{rad^2}}{sec^2} = \frac{kg\text{-}m^2}{sec^2} = N\text{-}m$$

$M = 45 \times 10^{-3}$ kg $\qquad I = 28125 \times 10^{-9}$ kg-m^2

$g = 9.81$ m/sec^2 $\qquad \omega = \frac{V_s}{r_s}, \frac{m}{sec}/\cancel{m} = rad/sec$

s, m	r_s, m x 10^{-3}	V_s, m/sec	ω_s, rad/sec	P.E. Mgs	T.E., $\frac{1}{2}MV_s^2$	R.E., $\frac{1}{2}I\omega_s^2$	K.E., T.E.+R.E.
0.0	13.00	0.000	0.00	0.000	0.0000	0.000	0.000
0.1	12.34	0.620	50.24	0.044	0.0086	0.035	0.044
0.2	11.68	0.838	71.75	0.088	0.0158	0.072	0.088
0.3	10.97	0.975	88.88	0.132	0.0214	0.111	0.132
0.4	10.22	1.060	103.72	0.177	0.0253	0.151	0.176
0.5	9.41	1.103	117.22	0.221	0.0274	0.193	0.220
0.6	8.52	1.107	129.93	0.265	0.0276	0.237	0.265
0.7	7.53	1.069	141.97	0.309	0.0257	0.283	0.309
0.8	6.39	0.981	153.52	0.353	0.0217	0.331	0.353
0.9	4.99	0.823	164.93	0.397	0.0152	0.383	0.398
1.0	3.00	0.528	176.00	0.441	0.0063	0.436	0.442


ENERGY CONVERSION
PLAYABLE YO-YO
0.5
0.4
0.3
0.2
0.1
0
ENERGY, N-m
P.E.=K.E.
R.E.
K.E.=T.E.+R.E.
T.E.
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
S, m

SUMMARY - PART I.

"LEVERAGE: STRING TO YO-YO" r_s/k_o WAS PRESENTED IN MONOGRAPH I; THERE, SEE pg. 4. HERE, IT REAPPEARS AT pg. 18 IN "LEVERAGE AND TENSION EFFECTS". THE APPARENT EQUALITY $J/Mr_s^2 = k_o^2/r_s^2$ IS AT LEAST TECHNICALLY INTERESTING. r_s/k_o, ITS SQUARE $(r_s/k_o)^2$, AND ITS RECIPROCAL SQUARE $(k_o/r_s)^2$ MAY HAVE APPLICATION AS "FIGURES OF MERIT" IN YO-YO DESIGN. "NEEDS A FUTURE LOOK.

GRAPHIC TRANSFORMATION OF VERTICAL VELOCITY-DISTANCE DATA TO VERTICAL VELOCITY-TIME DATA WAS BORN OF THIS AUTHOR'S INABILITY TO USE THE HIGHER CALCULUS ELLIPTIC INTEGRALS. HAPPILY, THE GRAPHIC METHOD, THOUGH LACKING A CERTAIN ELEGANCE, YIELDS RESULTS IN AGREEMENT WITH THOSE OF DR. BÜRGER. IT TOOK A WHILE TO FIND THE WAY, WHAT WITH DISTRACTIONS, YO-YO PRACTICE HOURS, AND LOSS OF KEY BRAIN CELLS THROUGH THESE MANY YEARS. "THE YO-YO: A TOY FLYWHEEL" HAS BEEN IN MY FILE FOR OVER TEN YEARS.

THE ARTICLE MIGHT BE FOUND IN MANY TECHNICAL LIBRARY BACK ISSUE COLLECTIONS; OR THE COMPLETE ISSUE (SEE REFERENCES: 1.) MAY YET BE AVAILABLE AT:

SIGMA XI, THE SCIENTIFIC RESEARCH SOCIETY
P.O. BOX 13975
RESEARCH TRIANGLE PARK NC 27709
800 243 6534 www.sigmaxi.org

PART II: A TRUE GYROSCOPE

WE DON'T KNOW WHO FIRST ATTACHED A TETHER TO A GROOVED DISK, FASHIONING A PRIMITIVE YO-YO. THE "SLEEPING" YO-YO WITH ITS STRING LOOPED AT THE AXLE CAME MORE RECENTLY, BUT MADE THE YO-YO A TRUE GYROSCOPE. NO FIRM MOUNTING, CAGES (GIMBALS), OR DRIVE MOTORS; ONLY THE DISKS, AXLE CONNECTION, AND STRING. RIM-WEIGHTED DESIGN, TRANSAXLE ASSEMBLY (ESPECIALLY THE BALL-BEARING), AND DISK-TO-STRING COUPLING DEVICES FOR GOOD RESPONSE BRING US TO THE CURRENT HAPPY "STATE OF YO".

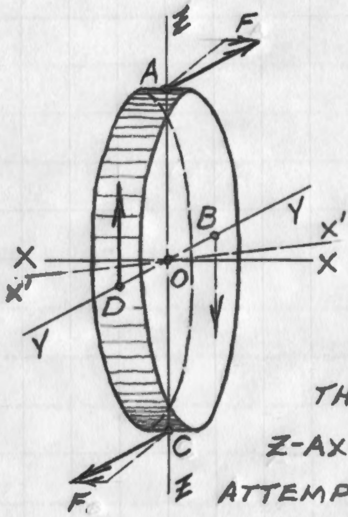
SO NOW WE HAVE IT – A SIMPLE GYROSCOPE, WITH ONLY STRING AND "ELBOW GREASE" FOR POWER, GIVING LONG SPINS AND STABLE PERFORMANCE IN INTRICATE MODERN TRICKS AND COMBINATIONS WITH RELIABLE RESPONSE.

THERE'S MORE. IN RECENTLY INTRODUCED "FREE-HAND" PLAY THE STRING POWERS THE YO-YO, THEN LEAVES THE HAND TO FLY WITH THE YO-YO IN NEW MOVES AND CATCHES.

MORE YET! IN "OFF-STRING" PLAY THE STRING POWERS, BUT IS NOT ATTACHED TO THE YO-YO. THE YO-YO FLIES FREE OF THE STRING, THEN BACK (OR TO ANOTHER'S STRING!) FOR PLAY AND CATCH. HERE, EVEN BRIEFLY SPINNING AND FLYING FREE IN MID-AIR, THE YO-YO IS A TRUE AND ELEGANT GYROSCOPE.

WORTHY OF STUDY.

GYROSCOPE PRECESSION
AN INTUITIVE PICTURE



VIEW THIS SOLID DISK AS ROTATING ABOUT ITS CENTER O IN THE DIRECTION OF ARROWS AT A, B, C, AND D IN THE Y-Z PLANE. ASSUME THE DISK IS FREE TO TILT ON THE Y-AXIS AND TURN ON THE Z-AXIS. A TORQUE (NOT SHOWN) ATTEMPTING TO TURN THE AXIS OF SPIN X-X IN THE X-Y PLANE TOWARD A NEW ANGULAR POSITION X'-X' ATTEMPTS TO TURN THE SOLID DISK ON THE Z-AXIS. THE SPINNING DISK INERTIALLY RESISTS, PRECESSING (i.e., TILTING) INSTEAD — CLOCKWISE ON THE Y-AXIS.

ELEMENTS OF THE DISK INDUCE FORCES F, RIGHT AT A AND LEFT AT C. THE ELEMENTS, RESISTING CHANGE IN DIRECTION OF HORIZONTAL FORWARD MOTION, INERTIALLY INDUCE THESE FORCES. AT B AND D ATTEMPTED TURNING OF THE DISK ON THE Z-AXIS HAS NO EFFECT ON THE VERTICAL FORWARD DIRECTION OF THE ELEMENTS.

EACH ELEMENT CONTRIBUTES MAXIMUM TILTING FORCE AT A (RIGHT), DECREASING TO ZERO AT B, INCREASING TO MAXIMUM AT C (LEFT), TO ZERO AGAIN AT D, . . . PRECESSING THE DISK SO LONG AS THE X-Y PLANE TORQUE IS ACTIVE.

RIGHT HAND RULE

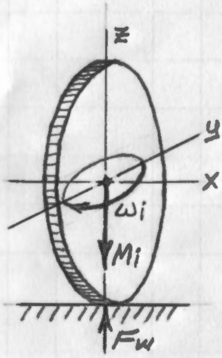
A BODY, ROTATING ABOUT A CENTRAL AXIS, WITH MOMENT OF INERTIA I AND INITIAL ANGULAR VELOCITY ω_i, HAS AN ANGULAR MOMENTUM (KINETIC ENERGY) $K.E. = \frac{1}{2} I \omega_i^2$ IN THE DIRECTION OF, AND IN THE PLANE OF, ROTATION. IT ALSO HAS AN INITIAL ANGULAR MOMENTUM VECTOR $M_i = I\omega_i$ DIRECTED ALONG THE AXIS OF ROTATION. M_i ACTS TO STABILIZE THE PLANE OF ROTATION.

A FORCE F_i APPLIED AT DISTANCE D FROM A BODY CENTER, AND AT 90° TO THAT RADIAL ARM, EXERTS A TORQUE *$T_i = F_i D$ IN THE DIRECTION OF F_i AND IN THE PLANE COMMON TO F_i AND D. TORQUE T_i APPLIES AN ANGULAR ACCELERATION TO THE BODY, CREATING A COMPANION TORQUE VECTOR *$\mathcal{T}_i = F_i D$ DIRECTED ALONG THE AXIS OF (IMPENDING) INITIAL ROTATION.

AMBIGUITY IN THE DIRECTION OF VECTOR M_i AND VECTOR $\mathcal{T}_i$ "ALONG THE AXIS . . ." IS BEST RESOLVED WITH THE "RIGHT HAND RULE" OF PHYSICAL ROTATION. IN THE NEXT TWO PAGES, THIS RULE IS STATED AND GRAPHICALLY DEFINED IN SPECIFIC SITUATIONS TO ILLUSTRATE THE DIRECTION OF EACH VECTOR.

*IN CALCULATIONS, $T_i = F_i D$ kg-m, BUT THE TORQUE VECTOR $\mathcal{T}_i = F_i D$ Newton-m OR $\frac{kg\text{-}m^2}{sec^2}$. BY DEFINITION, 1gm $= 9.81 \times 10^{-3}$ Newton.

ANGULAR MOMENTUM VECTOR



SPIN A COIN; ANY COIN. HERE, THE COIN IS SHOWN SPINNING CLOCKWISE AT INITIAL ANGULAR VELOCITY ω_i ABOUT VERTICAL AXIS Z. AT THE COIN BASE, FORCE F_W SUPPORTS AND EQUALS THE COIN WEIGHT.

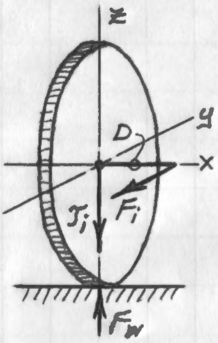
MOMENT OF INERTIA I_z AND ω_i DEFINE THE INITIAL KINETIC ENERGY K.E. OF THE SPIN AS $\frac{1}{2} I_z \omega_i^2$. FURTHER, AN INITIAL ANGULAR MOMENTUM VECTOR M_i EXISTS AS $I_z \omega_i$. THE RIGHT HAND RULE OF ROTATION PHYSICS DEFINES THE M_i DIRECTION; WRAP THE RIGHT HAND AROUND THE SPIN AXIS WITH FINGER TIPS IN THE DIRECTION OF ROTATION, THEN EXTEND THE THUMB PARALLEL TO THE SPIN AXIS TO INDICATE THE DIRECTION OF M_i – IN THIS CASE, DOWNWARD.

M_i IS THE MEASURE OF GYROSCOPIC STABILITY, MAINTAINING THE AXIS OF SPIN VERTICAL SO LONG AS ω_i IS UNDIMINISHED. THE COIN, SPINNING AS SHOWN, IS AN INEFFICIENT GYROSCOPE WITH AIR RESISTANCE AND SOME FRICTION AT THE BASE QUICKLY REDUCING ω_i AND M_i. THE SPIN AXIS SOON TILTS FROM THE VERTICAL AND, WITH EVER-INCREASING GRAVITY EFFECT, SPIRALS DOWN TO A STOP ON ONE FACE OR THE OTHER.

CLARIFYING UNITS OF MEASURE:

$$M_i = I_z \text{ kg-m}^2 \cdot \omega_i \frac{\cancel{\text{rad}}}{\text{sec}} = I_z \omega_i \frac{\text{kg-m}^2}{\text{sec}}$$

TORQUE VECTOR



SUPPORT A COIN; ANY COIN. HERE, THE COIN IS CONSTRAINED TO REVOLVE ABOUT VERTICAL AXIS Z; ROTATED BY AN INITIAL FORCE F_i ALWAYS ACTING PERPENDICULAR TO RADIAL LEVER ARM D, WITH F_i AND D BOTH IN THE HORIZONTAL X-Y PLANE.

AN INITIAL TORQUE T_i EXISTS AS F_iD EXERTING CLOCKWISE ANGULAR ACCELERATION α ABOUT VERTICAL AXIS Z. THIS APPLIED ACCELERATION PRODUCES A TORQUE VECTOR $\mathcal{T}_i$, WHERE $\mathcal{T}_i$ MUST BE EXPRESSED IN N-m UNITS. THE RIGHT HAND RULE APPLIES HERE TO $\mathcal{T}_i$ AS IT DID TO THE ANGULAR MOMENTUM VECTOR M_i. CLOCKWISE INITIAL TORQUE T_i DIRECTS THE TORQUE VECTOR $\mathcal{T}_i$ DOWNWARD AS SHOWN.

SO LONG AS THE EXTERNALLY APPLIED TORQUE T_i IS HELD CONSTANT, ANGULAR ACCELERATION α AND TORQUE VECTOR $\mathcal{T}_i$ REMAIN UNDIMINISHED AS WELL.

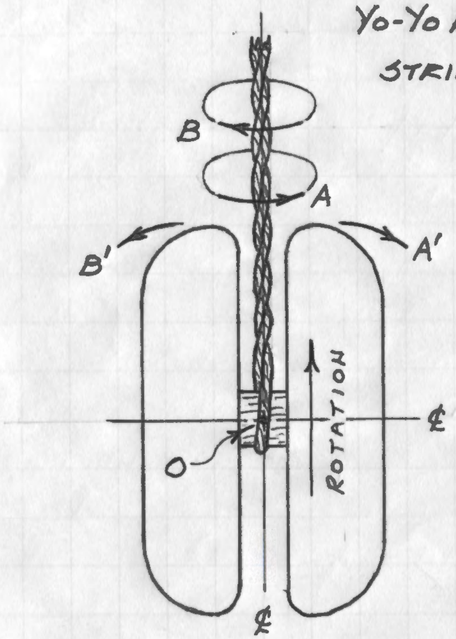
CLARIFYING UNITS OF MEASURE HERE:

$$T_i = F_i\ \text{kg} \cdot D\ \text{m} = F_i D\ \text{kg-m}$$

$$\mathcal{T}_i = F_i\ \text{N} \cdot D\ \text{m} = F_i\ \frac{\text{kg-m}}{\text{sec}^2} \cdot D\ \text{m}$$

$$\mathcal{T}_i = F_i D\ \frac{\text{kg-m}^2}{\text{sec}^2}$$

Yo-Yo Precession - Case 1
String Free of the Rims

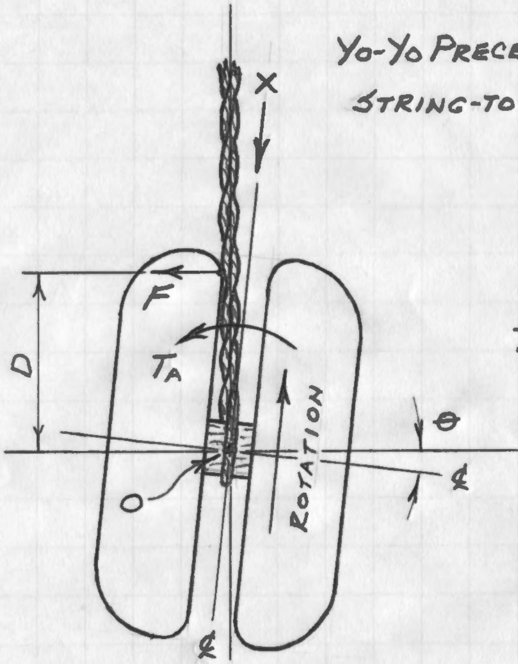


THE YO-YO IS SHOWN HANGING VERTICALLY AND SPINNING CLOCKWISE (AS VIEWED FROM THE RIGHT). TWISTING THE STRING WITH THE FINGERS CLOSE TO THE YO-YO AS AT A OR B APPLIES A TORQUE IN THE HORIZONTAL PLANE AT O, CAUSING YO-YO PRECESSION ABOUT O IN A VERTICAL PLANE. TWISTING:

- COUNTER-CLOCKWISE AS AT A PRECESSES (TILTS) THE YO-YO SLOWLY TO THE RIGHT (A').
- CLOCKWISE AS AT B PRECESSES THE YO-YO SLOWLY TO THE LEFT (B').

REVERSED YO-YO ROTATION YIELDS REVERSED PRECESSION FOR THE STATED STRING TWIST DIRECTIONS.

YO-YO PRECESSION – CASE 2
STRING-TO-RIM CONTACT



$T_A = F \cdot D$ gm-in

THE YO-YO, SPINNING CLOCKWISE (VIEWED FROM THE RIGHT) AND HANGING AT A TILT ANGLE θ, PRODUCES A LATERAL FORCE F WITH THE STRING IN CONTACT AT THE YO-YO RIM. FORCE F CAUSES PRECESSION TO OCCUR IN TWO DIRECTIONS:

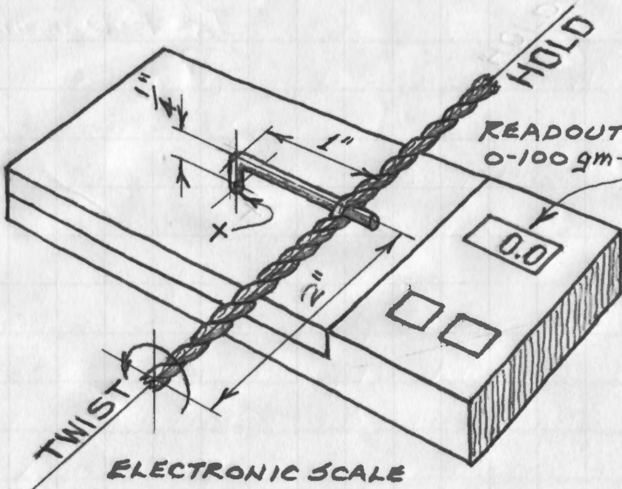
– TORQUE T_A ABOUT O IN THE LATERAL VERTICAL PLANE PRECESSES THE YO-YO ABOUT O, BUT COUNTER-CLOCKWISE AS VIEWED FROM X.

– FRICTION AT THE RIM FROM FORCE F PRODUCES A TORQUE ABOUT O COUNTER-CLOCKWISE (NOT SHOWN, VIEWED FROM X) PRECESSING THE YO-YO TO INCREASE THE TILT ANGLE θ.

REVERSED TILT REVERSES BOTH PRECESSION DIRECTIONS.

YO-YO STRING TORQUE

THE PICTURED APPARATUS MEASURES YO-YO STRING-TWIST TORQUE. VALUES FOR TORQUE VECTOR T ARE NEEDED WITH MOMENT OF INERTIA I AND ANGULAR VELOCITY ω; TO EVALUATE YO-YO STABILITY FROM CALCULATED GYROSCOPIC PRECESSION RATES (DEGREES/SECOND).



ELECTRONIC SCALE
STRING TORQUE TEST

AN ELECTRONIC GRAM SCALE (0-100 gm), 1½" LENGTH OF PAPER CLIP WIRE WITH A ¼" x 90° BEND, AND A STOCK UNUSED YO-YO STRING PROVIDE AN EFFECTIVE TEST. INSERT THE WIRE BETWEEN STRING STRANDS AND POSITION STRING AND WIRE AS SHOWN. THE RIGHT HAND IS USED TO "HOLD" AND THE LEFT IS USED TO "TWIST" THE STRING, WITH THE WIRE EXERTING FORCE AT POINT "X" NEAR THE SCALE STAGE CENTER. A SUGGESTED TEST PROCEDURE FOLLOWS.

TEST PROCEDURE:

1. "HOLD" THE STRING WITH THE RIGHT HAND A FOOT OR MORE FROM THE WIRE.
2. REST THE 1/4" WIRE TIP DOWN AT "X", THE SCALE STAGE CENTER.
3. WITH LIGHT STRING TENSION BETWEEN THE HANDS, CHECK FOR 0.0 gm AT THE READOUT.
4. "TWIST" AT THE LEFT, COUNTER-CLOCKWISE AS IN A YO-YO PRECESSION - CASE 1 TRIAL.
5. THE READOUT REGISTERS FORCE F AS EXERTED AT THE 1" LEVER ARM.

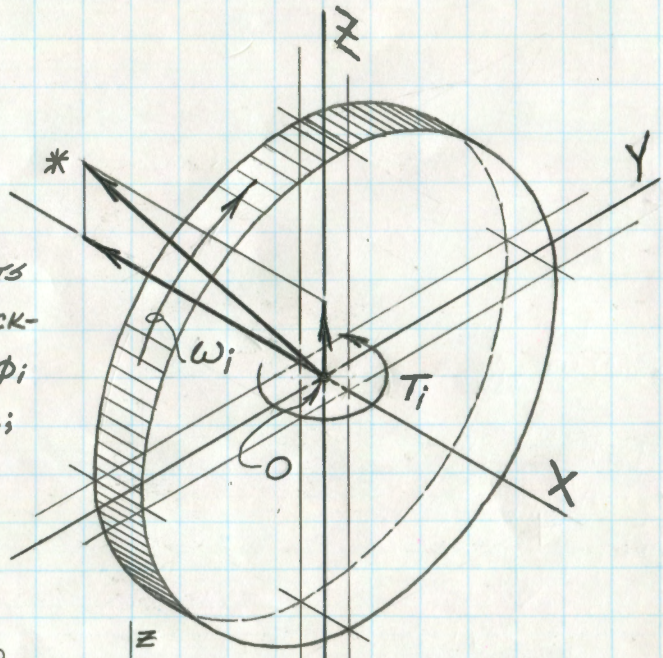
VALUES FOR F OBTAINED BY THE AUTHOR FELL IN THE RANGE $0.5 < F < 1.0$ gm. EXPECTED STRING TORQUE VALUES IN A CASE 1 TRIAL ARE:

$$0.5 < T_i < 1.0 \text{ gm-in}$$
$$= 12.7 \times 10^{-6} < T_i < 25.4 \times 10^{-6} \text{ kg-m}$$

HISTORICAL NOTE: IN THE EARLY 20TH CENTURY, PIERRE LECOMTE DU NOÜY, FAMED BIO-SCIENTIST AND PHILOSOPHER, DESIGNED A TENSIOMETER WITH A CALIBRATED PROTRACTOR DISK, TWISTING A PRECISION TORSION WIRE CARRYING A LEVER ARM TO LIFT A STANDARDIZED SUBMERSIBLE WIRE RING. THE DEGREE OF WIRE-TWIST TORQUE REQUIRED TO LIFT THE RING FREE OF A LIQUID SURFACE GAVE PRECISE MEASURE OF THE SURFACE TENSION. THE INSTRUMENT REMAINS IN WIDE USE TODAY.

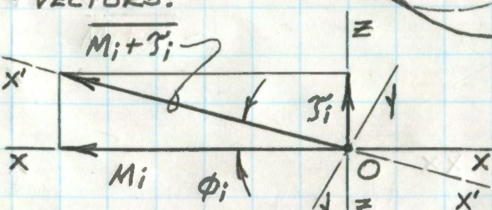
Yo-Yo Precession
Direction and Rate

View the solid disk below as equivalent to a playable Yo-Yo of weight M gm, moment of inertia I kg-m^2, rotating about center O as shown in the Y-Z plane at initial angular velocity ω_i rev/min under string-twist initial torque T_i gm-in about O as shown in the X-Y plane. By the right hand rule, ω_i produces initial angular momentum vector M_i on the X-axis, while T_i produces initial torque vector $\mathcal{T}_i$ on the Z-axis.



Note: $\mathcal{T}_i$ tilts the disk clockwise angle ϕ_i in plane X-Z;

* Vectors:



AS PICTURED AT LEFT (✱), TORQUE VECTOR $\mathcal{T}_i$ ACTS ON ANGULAR MOMENTUM VECTOR M_i TO TILT THE DISK AXIS X-X TO NEW POSITION X'-X' THROUGH AN ANGLE ϕ_i IN SOME PERIOD OF TIME. FOR THE GIVEN ω_i AND T_i DIRECTIONS ABOUT DISK CENTER O, THE YO-YO, EQUIVALENT TO THE DISK, PRECESSES CLOCKWISE ABOUT THE Y-AXIS; SEE ALSO YO-YO PRECESSION - CASE 1.

PRECESSION RATES CAN BE CALCULATED FROM THREE VARIABLES: MOMENT OF INERTIA J kg-m², A REASONABLE ANGULAR VELOCITY ω_i rev/min, AND A KNOWN PRECESSING TORQUE T_i kg-m. IN A PLAYABLE YO-YO TRIAL CASE, ASSUME:

$$J_i = 28125 \times 10^{-9} \text{ kg-m}^2$$

$$\omega_i = 4000 \text{ rev/min} \quad \text{(A MODERATE VALUE)}$$

$$\oplus\, T_i = 19.0 \times 10^{-6} \text{ kg-m}$$

$$\omega_i = 4000 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \cdot \frac{2\pi \text{ rad}}{\cancel{\text{rev}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \simeq 419 \frac{\text{rad}}{\text{sec}}$$

ANGULAR MOMENTUM VECTOR $M_I = J\omega_i$:

$$M_I = 28125 \times 10^{-9} \text{kg-m}^2 \cdot 419 \frac{\cancel{\text{rad}}}{\text{sec}} \simeq 11.8 \times 10^{-3} \frac{\text{kg-m}^2}{\text{sec}}$$

TORQUE VECTOR $\mathcal{T}_i$ IS T_i kg-m, BUT IN Newton-m:

$$\mathcal{T}_i = T_i \simeq 19.0 \times 10^{-6} \cancel{\text{kg}}\text{-m} \cdot \frac{1.0 N}{0.102 \cancel{\text{kg}}} \simeq 186 \times 10^{-6} N\text{-m}$$

$$\simeq 186 \times 10^{-6} \frac{\text{kg-m}^2}{\text{sec}^2}$$

⊕ SEE YO-YO STRING TORQUE FOR THIS AVERAGE.

THE PRECESSION RATE Ω VARIES DIRECTLY WITH APPLIED TORQUE AND INVERSELY WITH THE PRODUCT OF MOMENT OF INERTIA AND ANGULAR VELOCITY:

$$\Omega_i = \frac{T_i}{I\omega_i} = \frac{T_i}{M_i} = 186 \times 10^{-6} \frac{kg\text{-}m^2}{sec^2} \Big/ 11.8 \times 10^{-3} \frac{kg\text{-}m^2}{sec}$$

$$\Omega_i \simeq 15.8 \times 10^{-3} \frac{\cancel{rad}}{sec} \cdot \frac{360\, deg}{2\pi \cancel{rad}} \simeq 0.9 \frac{deg}{sec}$$

RECOGNIZE THAT THIS TRIAL YIELDS A PRECESSION RATE NEAR ONE DEGREE PER SECOND WITH (AS IN YO-YO PRECESSION-CASE 1 AND IN THE YO-YO STRING TORQUE TEST) THE FINGERS USED TO INDUCE HIGH VALUES OF PRECESSING TORQUE T_i. STRING TORQUE FOR THIS YO-YO IN A NORMAL HANGING "SLEEPER" IS ESTIMATED TO BE LESS THAN 10×10^{-6} kg-m; AT THE SAME 4000 RPM, THE PRECESSION RATE WOULD BE LESS THAN ONE-HALF DEGREE PER SECOND.

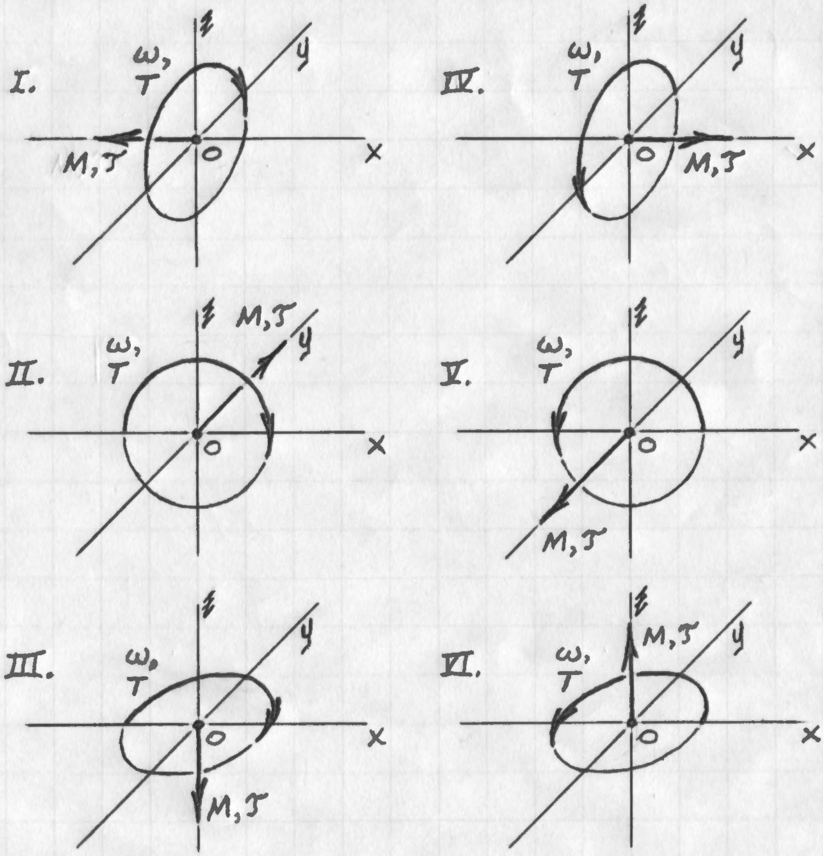
HELPFUL NOTE TO NOVICE PLAYERS:

THE "SLEEPER" MUST BE THROWN WITH ENERGY DEVELOPING A FEW THOUSAND RPM TO ACHIEVE GOOD SPIN DURATION AND CONTROL AT LOW STRING TORQUE INDUCED PRECESSION RATES.

YO-YO PRECESSION - CASE 2 SHOWS THE IMPORTANCE OF THROWING THE YO-YO ACCURATELY, KEEPING SPINNING SIDES FREE OF THE STRING. FRICTION-INDUCED TORQUES (TWO!) AT A RIM TURN THE YO-YO ON THE STRING AXIS, WHILE ALSO TILTING IT FORWARD. CONTROL IS QUICKLY LOST; HERE, STRING TWIST TORQUE IS A MINOR FACTOR.

VECTOR CARDINAL DIRECTIONS I – VI
FOR THE RIGHT HAND RULE

CLOCKWISE: COUNTER-CLOCKWISE:



UNITS:

ω = ANGULAR VELOCITY, $\frac{rad}{sec}$

T = APPLIED TORQUE, $F\,kg \cdot D\,m = FD\,kg\text{-}m$

M = ANGULAR MOMENTUM VECTOR,

$= I\,kg\text{-}m^2 \cdot \omega \frac{\cancel{rad}}{sec} = I\omega\,\frac{kg\text{-}m^2}{sec}$

$\mathcal{T}$ = TORQUE VECTOR, F Newton $\cdot\, D\,m$

$= F\,\frac{kg\text{-}m}{sec^2} \cdot D\,m = FD\,\frac{kg\text{-}m^2}{sec^2}$

SUMMARY - PART II

ANGULAR VELOCITY THOUSANDS OF TIMES GREATER THAN PRECESSION RATE IS REQUIRED FOR GOOD GYROSCOPE STABILITY. THE RATIO OF 4000 RPM TO ABOUT 1 DEGREE PER SECOND (pgs. 35 AND 36) AT 24000 TO 1 DEFINES THE "PLAYABLE YO-YO" AS A GOOD GYROSCOPE. A YO-YO WITH HALF THE MOMENT OF INERTIA WOULD EXHIBIT TWICE THE PRECESSION RATE AT 4000 RPM, AND THE SAME PRECESSION RATE (1 deg/sec) AT 8000 RPM, AT 19.6×10^{-6} kg-m STRING TWIST TORQUE (pg. 35).

IN THE RECENT DECADE, NEW YO-YO DESIGNS HAVE PROLIFERATED. STRING CAPS ARE WIDER, RIM-WEIGHTED AND HEAVIER YO-YOS ARE POPULAR, BALL-BEARING AXLE STRUCTURES WITH SPINNING SIDES-TO-STRING COUPLING MEANS (FOR GOOD RESPONSE AND CONTROL) ARE IN GREAT DEMAND. LONG SPIN DURATIONS WITH MUCH-IMPROVED GYROSCOPIC STABILITY ARE NOW COMMON TO MOST "HIGH-TECH" YO-YOS. PLAYERS HAVE RESPONDED WITH A DAZZLING ARRAY OF NEW MOVES, TRICKS, AND MODES OF PLAY, MANY OF WHICH WERE NOT POSSIBLE WITH EARLIER WOOD OR PLASTIC "FIXED AXLE" DESIGNS.

LOOK FOR EVEN GREATER ADVANCES IN YO-YO DESIGN AND PLAYER SKILLS IN THE NEXT WAVE OF YO-YO POPULARITY. MAY IT ARRIVE SOON!

* * *

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